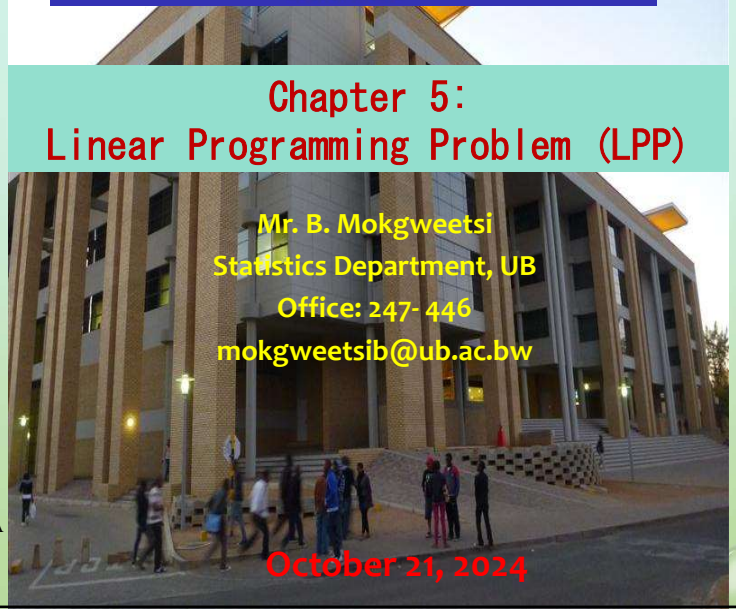
STA 101: Mathematics for Business and Social Sciences I



Chapter 5: Linear Programming Problem (LPP)

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STA 101 TEST 3

DATE:03 NOVEMBER 2024

TIME : 1PM-3PM

VENUES:240/11

CHAPTER 5:
Linear Programming Problem

Motivation

Consider the hawker selling airtime for Orange, Mascom, and BTC with the objective to maximizing profit.

P1500

Max Profit $Z = P_1X + P_2Y + P_3\omega$ (decision variable)

Budget constraints

$C_1X + C_2Y + C_3\omega \leq 1500$ (constraint)

* graph inequalities *

x y ω

800 300 400

Min ₹ =

Explanation for the Hawker:
This linear programming model will help you maximize your profit from selling Orange, Mascom, and BTC airtime vouchers. By considering the demand for each type of airtime and your available budget, you can determine the optimal number of vouchers to sell for each provider to achieve the highest profit.

5: LINEAR PROGRAMMING PROBLEM

Linear programming (LP) is a method to achieve the best outcome (such as **maximum profit** or **minimize cost**) in a mathematical model whose requirements are represented by linear relationships.

Programming problems are generally concerned with the allocation of scarce resources. These resources include labour, capital, machines, material, etc.

5: LINEAR PROGRAMMING PROBLEM

- Linear Programming Problem (**LPP**) is one of the types of programming problem. It is a tool for solving optimization (minimization or maximization) problems.

- LPP is applied in various areas such as industries, banking, education, forestry etc.

5: LINEAR PROGRAMMING PROBLEM

Summarized Example 1 of LPP

Objective fn

Maximize $Z = 12x + 8y$

Subject to "graph"

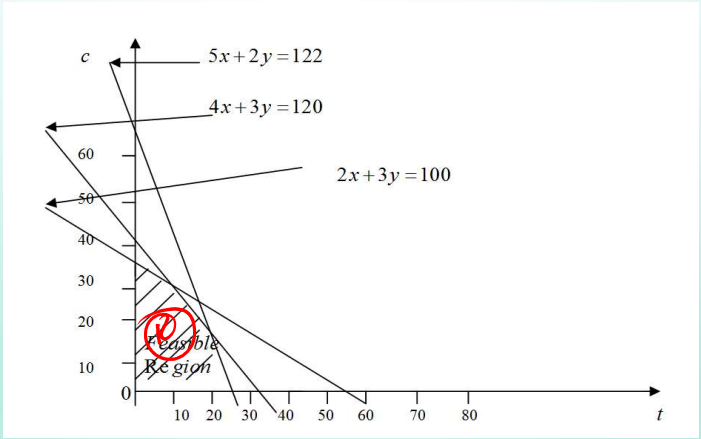
① $5x + 2y \leq 122$
② $2x + 3y \leq 100$
③ $4x + 3y \leq 120$

} constraints

NB: $x \geq 0$ and $y \geq 0$. } non-neg constraints

5: LINEAR PROGRAMMING PROBLEM

Graphical Solution to LP Problems



5: LINEAR PROGRAMMING PROBLEM

Summarized Example 2 of LPP

x and y
x_1 and x_2

Maximize

objective fn

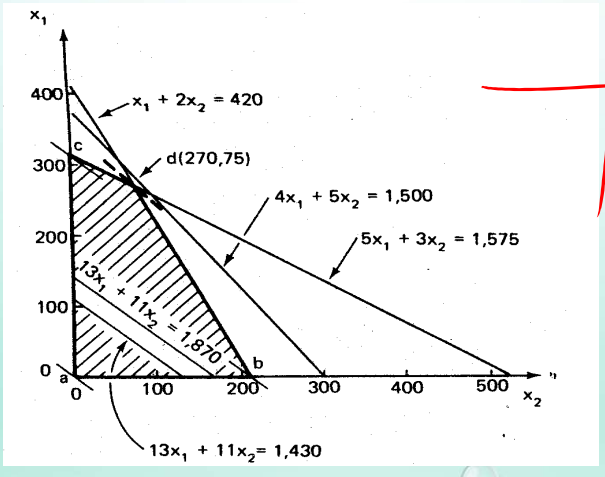
$$Z = 13x_1 + 11x_2$$

subject to:

$$\begin{aligned} 4x_1 + 5x_2 &\leq 1500 \\ 5x_1 + 3x_2 &\leq 1575 \\ x_1 + 2x_2 &\leq 420 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

5: LINEAR PROGRAMMING PROBLEM

Graphical Solution to LP Problems



5.1: DEFINITION OF TERMS AND STATEMENT OF L.P.P

i. Decision variables

A set of variables which are present in the objective function and the constraints of an L.P.P are called **decision variables**.

The decision variables are denoted by

$$x_1, x_2, \ldots, x_n.$$

5.1: DEFINITION OF TERMS AND STATEMENT OF L.P.P

ii. Objective function:

A linear function of the decision variable which is required to be **optimized** (either maximized or minimized) is called an objective function of an L.P.P and is denoted by $Z = c_1X + c_2Y$

$z = 3x + 8y$

max
min

$$Z = c_1X_1 + c_2X_2 + \ldots + c_nX_n$$

where the constants

$$c_1, c_2, \ldots, c_n$$

are known as technological coefficients.

5.1: DEFINITION OF TERMS AND STATEMENT OF L.P.P

iii. Constraints:

A set of linear equations/inequalities of the decision variables is called constraints.

While solving a L.P.P these constraints are **not to be violated.**

Maximize

$$Z = 13x_1 + 11x_2$$

subject to:

$$\begin{aligned} 4x_1 + 5x_2 &\leq 1500 \\ 5x_1 + 3x_2 &\leq 1575 \\ x_1 + 2x_2 &\leq 420 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

- Objective function
- Constraints:

iv. Non-negative restrictions:

In a L.P.P, the decision variables

$$x_1, x_2, \ldots, x_n$$

have to be non negative. In other words, we must have

$$x_1 \geq 0,$$
$$x_2 \geq 0,$$
$$\vdots$$
$$x_n \geq 0$$

This requirement is called **non-negative restrictions**.

5: LINEAR PROGRAMMING PROBLEM

v. Statement of a L.P.P.:

A linear programming problem is a mathematical problem that is concerned with **optimizing** (maximizing or minimizing) a linear function of the decision variables subject to a system of linear constraints (equalities or inequalities) and non-negativity restrictions of the decision variables.

5: LINEAR PROGRAMMING PROBLEM

vi. Solution of a L.P.P:

Solving a L.P.P. means finding a set of decision variables which optimize the objective function and satisfy the given set of constrains and non-negativity restrictions.

5.3 Formulation of a L.P.P

The steps involved in formulation of a L.P.P are the following:

a)Identify the decision variables $x_1, x_2, \ldots, x_n$.

b)Formulate the objective function

$z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ which is to be optimized

c) Identify the set of constrains which are not to be violated and the non-negativity restrictions.

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YOUTUBE VIDEOS

https://www.youtube.com/watch?v=pP0Qag694Go

https://www.youtube.com/watch?v=0TD9EQcheZM&t=236s

STEPS INVOLVED IN SOLVING A L.P.P IN TWO VARIABLES BY GRAPHICAL METHOD

We go through the following steps while solving a L.P.P in two variables by the graphical

STEP 1 Formulate the L.P.P using the given information

STEP 2 Represent the constraints of the L.P.P. in the (x-y) plane using a graph.

STEP 3 Identify the feasible region and the vertices of the convex region corresponding to the feasible region

STEP 4 Evaluate the value of the objective function at these vertices

STEP 5 Find the optimum (maximum or minimum) value of the objective function and the value of the corresponding decision variable at this vertex.

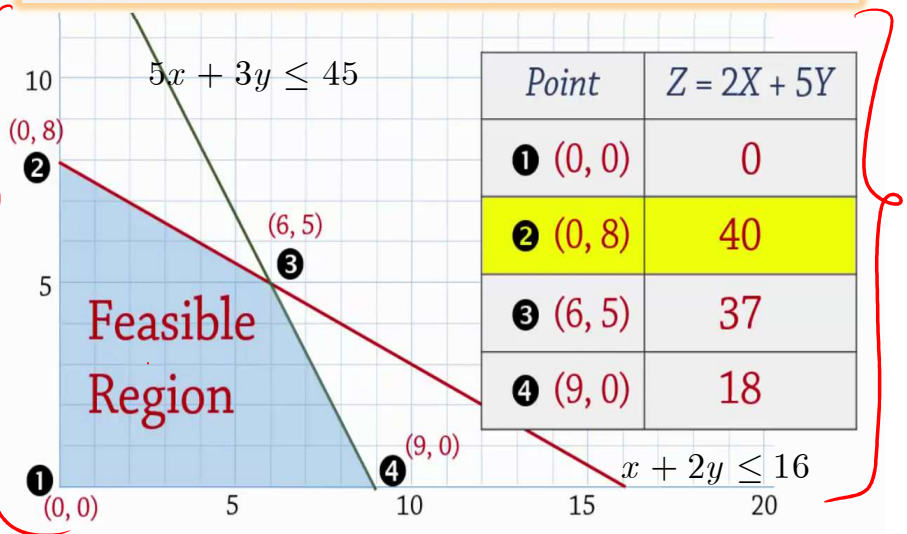
LPP:FURNITURE PROBLEM

A furniture company makes and sells tables and chairs. Each table requires 5 board-feet of oak, 2 board-feet of pine, and 4 labour-hours. Each chair requires 2 board-feet of oak, 3 board-feet of pine and 3 labour hours. There are 130 board feet of oak, 102 board feet of pine and 120 labour hours available for the upcoming week. The profit per table is P12 while profit per chair is P8.

max $Z = 12x + 8y$

i) Formulate the furniture company production problem as a linear programming problem.
ii) Draw a graph showing the feasible region.
iii) Using the graph, find the number tables and chairs should the company make during the next week so as to maximize its profit. What is the profit at this production level?

5: Linear Programming Problem



Point	Z = 2X + 5Y
❶ (0, 0)	0
❷ (0, 8)	40
❸ (6, 5)	37
❹ (9, 0)	18

http://cbom.atozmath.com/CBOM/Simplex.aspx?q=gm

EXAMPLE 1

$$\begin{aligned} \text{Max} \quad & 2X + 5Y \\ \text{s.t.} \quad & \\ & X + 2Y \leq 16 \\ & 5X + 3Y \leq 45 \\ & X, Y \geq 0 \end{aligned}$$

Objective:

Solve maximization LP model graphically

$$\begin{aligned} \text{Max} \quad & 2X + 5Y \\ \text{s.t.} \quad & \\ & X + 2Y \leq 16 \\ & 5X + 3Y \leq 45 \\ & X, Y \geq 0 \end{aligned}$$

Linear Programming (LP) model

$$\text{Max} \quad 2X + 5Y$$
$$\text{s.t.}$$
$$X + 2Y \leq 16$$
$$5X + 3Y \leq 45$$
$$X, Y \geq 0$$

$X, Y \rightarrow$ *Decision Variables*

$$\text{Max} \quad 2X + 5Y \rightarrow \textit{Objective Function}$$
$$\text{s.t.}$$
$$X + 2Y \leq 16$$
$$5X + 3Y \leq 45$$
$$X, Y \geq 0$$

Max $2X + 5Y$

s.t.

$X + 2Y \leq 16$

$5X + 3Y \leq 45$

$X, Y \geq 0$

graph

Constraints

Max $2X + 5Y$

s.t.

$X + 2Y \leq 16$ ...C1

$5X + 3Y \leq 45$ ...C2

always state

$X, Y \geq 0$

Non-negativity constraints

Plot the constraints

Max $2X + 5Y$

s.t.

$X + 2Y \leq 16$ ...C1

$5X + 3Y \leq 45$ ...C2

$X, Y \geq 0$

$x + 2y = 16$

$x + 2(0) = 16$

$x = 16$

$X + 2Y \leq 16$

X	Y	
0	8	(0,8)
16	0	(16,0)

$x + 2y = 16$

$0 + 2y = 16$

$\frac{2y}{2} = \frac{16}{2}$

$y = 8$

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Max $2X + 5Y$

s.t.

$X + 2Y \leq 16$ ...C1

$5X + 3Y \leq 45$ ...C2

$X, Y \geq 0$

$X + 2Y \leq 16$

X	Y
0	8
16	0

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Max $2X + 5Y$

s.t.

$X + 2Y \leq 16$ ...C1

$5X + 3Y \leq 45$ ...C2

$X, Y \geq 0$

$5x + 3y = 45$

$X + 2Y \leq 16$

X	Y	
0	8	(0,8)
16	0	(16,0)

$5X + 3Y \leq 45$

X	Y	
0	15	(0,15)
9	0	(9,0)

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Max $2X + 5Y$

s.t.

$X + 2Y \leq 16$ ...C1

$5X + 3Y \leq 45$ ...C2

$X, Y \geq 0$

$X + 2Y \leq 16$

X	Y	
0	8	(0,8)
16	0	(16,0)

$5X + 3Y \leq 45$

X	Y	
0	15	(0,15)
9	0	(9,0)

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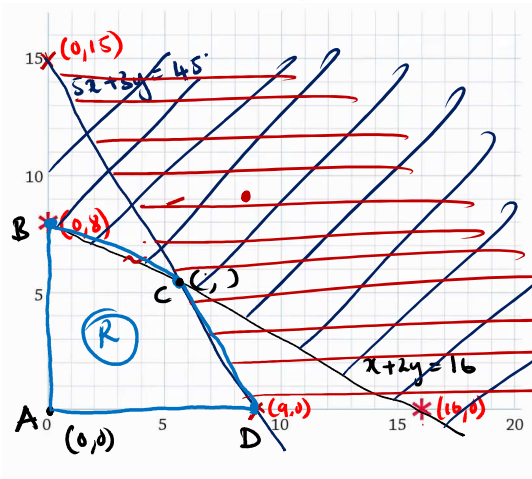
Shade unwanted region
$x + 2y = 16$
$5x + 3y \leq 45$

$x + 2y \leq 16$
$0 + 2(0) \leq 16$
$0 < 16$



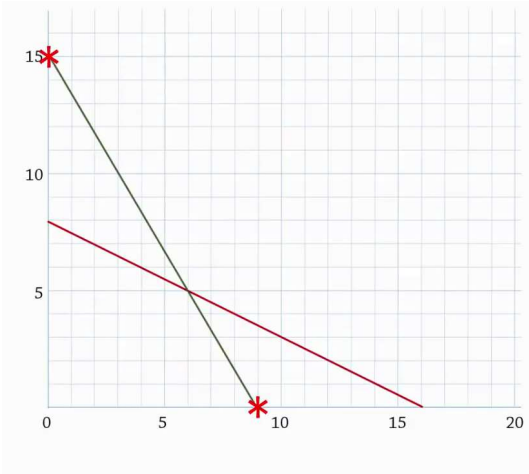
$X + 2Y \leq 16$

X	Y
0	8
16	0

$5X + 3Y \leq 45$

X	Y
0	15
9	0

$5x + 3y \leq 45$
$5(0) + 3(0) \leq 45$
$0 < 45$



$X + 2Y \leq 16$

X	Y
0	8
16	0

$5X + 3Y \leq 45$

X	Y
0	15
9	0

$X + 2Y \leq 16$

X	Y
0	8
16	0

$5X + 3Y \leq 45$

X	Y
0	15
9	0

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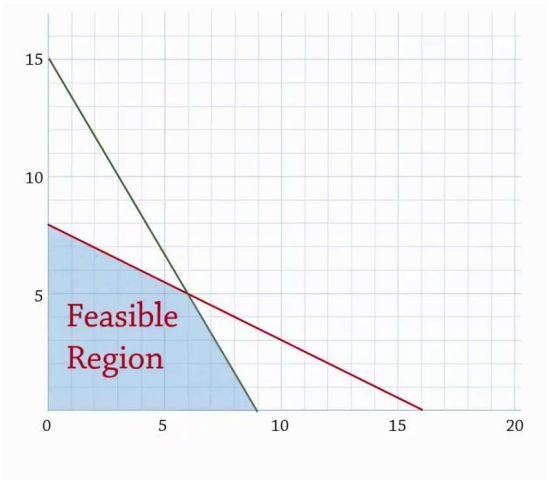

$X + 2Y \leq 16$

X	Y
0	8
16	0

$5X + 3Y \leq 45$

X	Y
0	15
9	0

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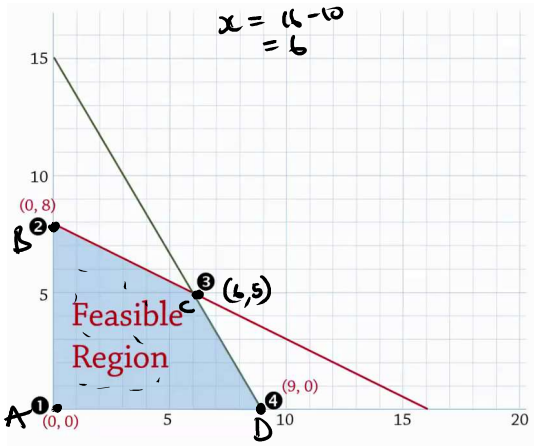
$x + 2y = 16$
$x + 2(5) = 16$
$x + 10 = 16$
$x = 16 - 10$
$= 6$



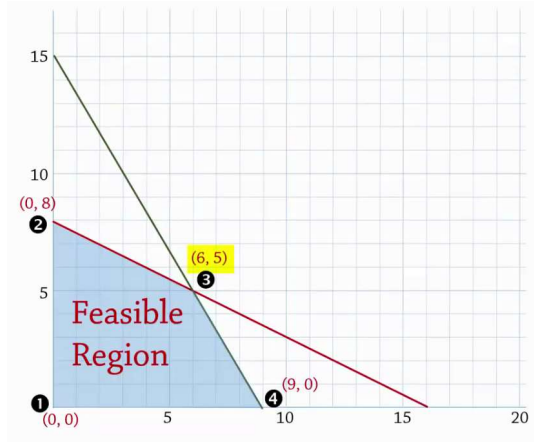
$(1X + 2Y = 16)$ ...C1 $\times 5$

$(5X + 3Y = 45)$ ...C2 $\times 1$

$5x + 10y = 80$
$-\ 5x + 3y = 45$
$0 + 7y = 35$

$\frac{7y}{7} = \frac{35}{7}$

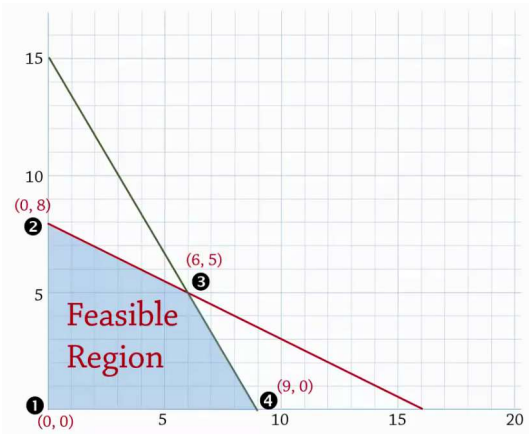
$y = 5$



X + 2Y = 16 ...C1
5X + 10Y=80
5X + 3Y = 45 ...C2

7Y = 35
Y = 5

X + 2(5) = 16
X + 10 = 16
X = 16 - 10
X = 6



$X + 2Y = 16$ ...C1

$5X + 10Y = 80$

$5X + 3Y = 45$ ...C2

$7Y = 35$

$Y = 5$

$X + 2(5) = 16$

$X + 10 = 16$

$X = 16 - 10$

$X = 6$

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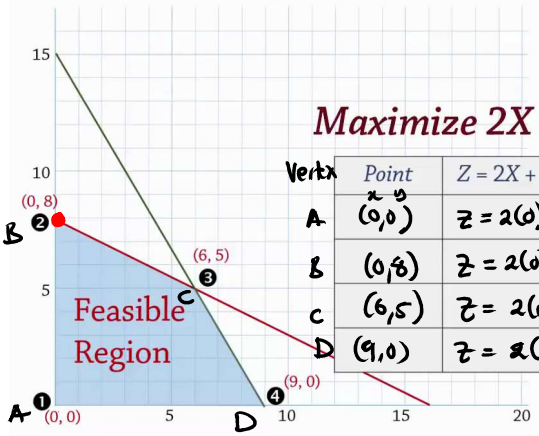

Maximize 2X + 5Y

Vertex	Point (x, y)	Z = 2X + 5Y
A	(0,0)	$Z = 2(0) + 5(0) = 0$
B	(0,8)	$Z = 2(0) + 5(8) = 40$
C	(6,5)	$Z = 2(6) + 5(5) = 37$
D	(9,0)	$Z = 2(9) + 5(0) = 18$

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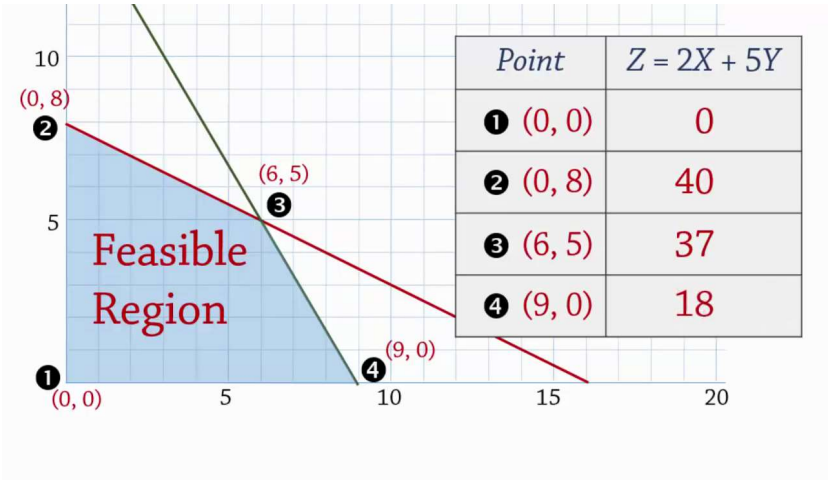

Point	$Z = 2X + 5Y$
❶ (0, 0)	0
❷ (0, 8)	40
❸ (6, 5)	37
❹ (9, 0)	18

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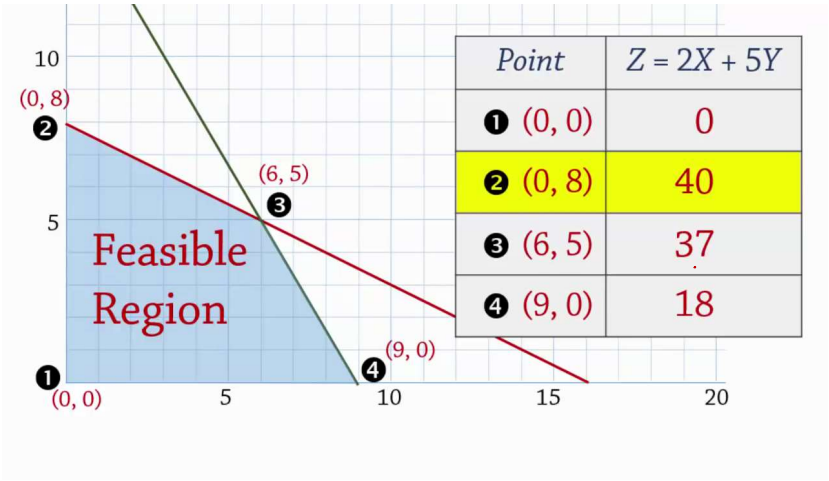

Point	$Z = 2X + 5Y$
❶ (0, 0)	0
❷ (0, 8)	40
❸ (6, 5)	37
❹ (9, 0)	18

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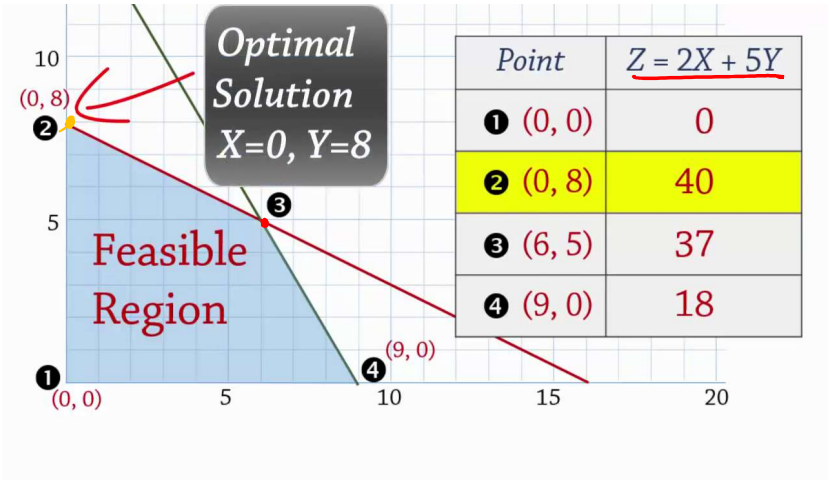
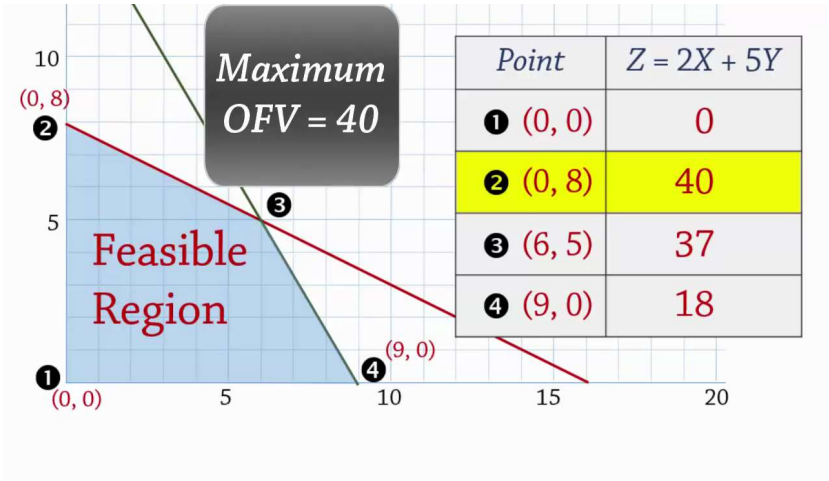

Optimal Solution X=0, Y=8
Feasible Region
(0, 8)
(9, 0)
(0, 0)
10
5
15
20
Point
Z = 2X + 5Y
❶ (0, 0) 0
❷ (0, 8) 40
❸ (6, 5) 37
❹ (9, 0) 18


Maximum OFV = 40
Feasible Region
(0, 8)
(9, 0)
(0, 0)
10
5
15
20
Point
Z = 2X + 5Y
❶ (0, 0) 0
❷ (0, 8) 40
❸ (6, 5) 37
❹ (9, 0) 18

EXAMPLE 2

objective fn

$$\max \quad P = 140x + 100y$$

profit

subject to

graph

$$c_1 \quad 5x + 2y \leq 420$$

$$c_2 \quad 2x + 3y \leq 300$$

non negative constraints

$$x \geq 0,\ y \geq 0$$

Shade unwanted region !!